Carderock Division
Naval Surface Warfare Center
9500 MacArthur Blvd.
West Bethesda, MD 20817-5700

NSWCCD-70-TR-2000/072 April 2000
Signatures Directorate
Research and Development Report

Induced Damping by a *Nearly* Continuous Distribution of *Nearly* Undamped Oscillators: Linear Analysis

by

G. Maidanik

Approved for public release; Distribution is unlimited

NSWCCD-70-TR-2000/072 Induced Damping by a *Nearly* Continuous Distribution of *Nearly* Undamped Oscillators: Linear Analysis

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS None
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release. Distribution is unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NSWCCD-70-TR—2000/072	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Carderock Division, NSWC 6b. OFFICE SYMBOL *(if applicable)* Code 7030	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS *(City, State, and ZIP Code)* 9500 MacArthur Blvd. West Bethesda, MD 20817-5700	7b. ADDRESS *(City, State, and ZIP Code)*
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL *(if applicable)*	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS *(City, State, and ZIP Code)*	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*

Induced Damping by a Nearly Continuous Distribution of Nearly Undamped Oscillators: Linear Analysis

12. PERSONAL AUTHOR(S)

G. Maidanik

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT *(Year, Month, Day)*	15. PAGE COUNT
Research & Development	FROM 03/01/00 TO 04/28/00	2000 April 28	43

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*

Oscillators
Damping
Pressure Transducers
Velocity Transducer

18. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

It has been claimed that induced damping can be derived from the coupling of a continuous distribution of undamped oscillators. This claim stems from the fact that the contribution to the damping of a master oscillator by a coupled set of continuously distributed satellite oscillators, is independent of the loss factors of the oscillators in this set. (The distribution is with respect to the frequency of resonance of the satellite oscillators in the set). In the determination of the induced damping, the transition from a discrete-to-a continuous distribution, however, cannot be achieved without the imposition of modal overlap on the distribution and on the inherent damping of the satellite oscillators. It is this imposition that ensures that the contribution to the damping by these satellite oscillators is intuitively real. The imposition forbids equating the loss factors of the satellite oscillators to zero just because their contribution to the surrogate damping -- the induced damping -- of the host master oscillator to which they are coupled, is independent of these loss factors.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT [X] UNCLASSIFIED/UNLIMITED [ ] SAME AS RPT. [ ] DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Maidanik, G.	22b. TELEPHONE (Including Area Code) (301) 227-1292 22c. OFFICE SYMBOL NSWCCD 7030

Contents

Preface

The report entitled "Vibration Damping by a *Nearly* Continuous Distribution of *Nearly* Undamped Oscillators," NSWCCD-70-TR-1999-120 (April 1999) was submitted as a paper to be published in the Journal of Sound and Vibration. Two reviewers set upon this paper and I was asked to shorten and revise some of the arguments. Unfortunately, the reviewers failed to understand the gist of the paper, in part I suppose the blame is mine. Consequently, I decided to reiterate a major argument in the submitted paper and make it the center piece of the revised version. The letter I sent with the revised manuscript to the Chief American Editor of this Journal; Professor Soedel, is enclosed for your perusal. I thought some readers of the present report may enjoy a reprint of this revised version of the manuscript.

Dear Professor Soedel:

Enclosed is a condensed and revised version of the paper previously submitted for publication under the title "Vibration Damping by a *Nearly* Continuous Distribution of *Nearly* Undamped Oscillators." Since that submission, the paper was reviewed by two reviewers and I am in receipt of their reviews. The first reviewer admits that he is long-winded and, in addition, he is patronizing. For the long-windedness, he is forgiven. I am not going to review either his review or that of the second reviewer. It suffices to say, in response, that the issue has not been centered about whether an integral can or cannot be performed. The issue is more subtle than that: Has the integral been obtained legitimately and does that result make sense? The revised manuscript, enclosed herein, tries to make this very point clearer than the original manuscript did. This was particularly requested by the second reviewer. I also omitted, for now, the energetics of this complex structure. These will be submitted, in due course under separate cover. Thus, it may be possible that the reviewers have positively contributed. May I, nonetheless, apologize to the authors of Reference 1 (now Reference (5)). It was not my intention to torpedo their paper or imply that they are wrong. I said it then, and I repeat it now: Once the auxiliary condition of modal overlap is addressed and satisfied, References 1-6 are largely validated. I do believe, however, that this condition is implicitly made in all these references and, therefore, setting the loss factors of the satellite oscillators a priori to zero, is a violation of what has already been implied.

I have no objection to this enclosed version of the paper being published as a *letter*, although, I would prefer a *paper*, subject to acceptance in the first place. The oversight of the auxiliary modal overlap condition has been circulating about for more than five years and, in spite of oral pleas, it has gone unnoticed. The convulsion over "where the energy went" is still an issue for some. It is time to lay it to rest. I hope this paper will do just that.

Thank you for your interest and I await your decision in this matter.

Sincerely yours,

G. Maidanik

THIS PAGE INTENTIONALLY LEFT BLANK

Abstract

It has been claimed that induced damping can be derived from the coupling of a continuous distribution of undamped oscillators. This claim stems from the fact that the contribution to the damping of a master oscillator by a coupled set of continuously distributed satellite oscillators, is independent of the loss factors of the oscillators in this set. (The distribution is with respect to the frequency of resonance of the satellite oscillators in the set). In the determination of the induced damping, the transition from a discrete-to-a continuous distribution, however, cannot be achieved without the imposition of modal overlap on the distribution and on the inherent damping of the satellite oscillators. It is this imposition that ensures that the contribution to the damping by these satellite oscillators is intuitively real. The imposition forbids equating the loss factors of the satellite oscillators to zero just because their contribution to the surrogate damping -- the induced damping -- of the host master oscillator to which they are coupled, is independent of these loss factors.

Introduction

A central theme to a few publications is that a set of satellite oscillators attached to a master oscillator contributes to the damping of the master oscillator even if all the loss factors of the satellite oscillators in the set, are equal to zero [1-6] [cf. Fig. 1.] The satellite oscillators are numerable and their resonance frequencies are distributed on both sides of the resonance frequency of the master oscillator. The conclusion that a set of lossless satellite oscillators can, nonetheless, contribute to the damping of a master oscillator, has been supported by analyses that a priori assume the satellite oscillators to posses zero loss factors [4, 5]. In contrast to the analysis presented here, some of the analyses; e.g., the analysis in Reference 5, are conducted in the time domain [3]. With respect to the material presented in this paper, whether the analysis is conducted in the time domain or in the frequency domain, is considered to be merely a matter choice, notwithstanding that in the case of a closely packed distribution of resonance frequencies of the satellite oscillators, the relaxation times involved may be quite long. In this case, to relate the results in the time domain to those in the frequency domain may require long times to capture the entire data [3].

In this present paper, a fundamental initial oversight that beset all these publications is brought to the attention of the reader. Admittedly, once that oversight is addressed, the remaining analyses and arguments in all these publications are largely validated; e.g., validated is the argument that specifically defined measure of the damping that is provided to the host master oscillator by a distribution of nearly lossless oscillators, is independent of the loss factors that account for the dampings in these oscillators. This independence does not, however, imply that individual loss factors that are equal to zero are admissible; indeed, individual loss factors that are not equal to zero are an essential condition on the validity of the

analysis in the first place [2,7]. Moreover, the analysis of nearly continuous distribution of nearly undamped satellite oscillators brings insights into the manner by which the satellite oscillators contribute damping to the mechanical system comprising a hosting master oscillator and these satellite oscillators. In part, these insights are obscured in an analysis that is subjected a priori to limiting asymptotic conditions of negligibility and continuity; notwithstanding that such impositions may require careful consideration if apparent singular behaviors of the mechanical system are to be avoided. However, detailed considerations of this kind are to be presented under separate cover [8]. Here, as stated, the focus is on the linear equations of motion that describe the response of a complex structure comprising a master oscillator and a set of coupled satellite oscillators. The induced loss factor in the damping of the master oscillator is of particular interest. It is argued that the determination of the induced loss factor is readily achieved by replacing integration for the summation over satellite oscillators. However, such a replacement demands the obedience of an auxiliary relationship. This relationship imposes that a modal overlap condition be satisfied. A satisfied condition of modal overlap states that if the loss factors of the satellite oscillators are made to approach infinitesimal values, the number of these satellite oscillators must approach infinity at least at the same rate. This condition, then, precludes setting the loss factors of the satellite oscillators equal to zero a priori and in that sense, the imposition of the descriptive adjective "nearly" is meaningful. Conversely, it is asserted that adhocly replacing a summation by an integration renders the condition of modal overlap implicitly and a priori satisfied. The significance of these statements is that if the condition of modal overlap is satisfied, collectively the satellite oscillators possess enough damping to explain the physical presence of the induced loss factor. Then, inventing exotic mechanisms to account for the induced damping becomes moot.

I. The Formalism of a Discrete and of a Continuous Distribution of Resonance Frequencies.

Consider the mechanical system comprising a master oscillator, with mass (M) and stiffness (K), that is coupled to a set of satellite oscillators. This mechanical system is sketched in Fig. 1 [1-6]. The $(r)th$ satellite oscillator is defined by a mass (m_r) and a stiffness (k_r). The damping is assumed to be associated with the stiffness elements

$$K = K_o(1+i\eta_o) \; ; \qquad (K_o / M) = \omega_o^2 \quad , \tag{1a}$$

$$k_r = k_{or}(1+i\eta_r) \; ; \qquad (k_{or} / m_r) = \omega_r^2 \quad , \tag{1b}$$

where (η_o) and (η_r) are loss factors. Each of these loss factors characterizes an individual oscillator and quantifies the dissipation that the oscillator, in isolation, can handle when externally driven. The linear equations of motion of the master oscillator in situ and of a typical satellite oscillator in situ are

$$[(i\omega M)+(K / i\omega)]V_o(\omega)+\sum_1^R (k_r / i\omega)[V_o(\omega)-V_r(\omega)] = P_e(\omega) \quad , \tag{2}$$

$$(i\omega\, m_r)\; V_r(\omega) + (k_r / i\omega)\; [V_r(\omega) - V_o(\omega)] = 0 \quad , \tag{3}$$

respectively, where $V_o(\omega)$ and $V_r(\omega)$ are the responses of the mass (M) of the master oscillator and of the mass (m_r) of the $(r)th$ satellite oscillator, respectively, (R) is the

number of satellite oscillators that are coupled to the master oscillator and $P_e(\omega)$ is the drive that is assumed applied externally to the master oscillator; the satellite oscillators are not driven externally [7]. [cf. Fig. 1.] The satellite oscillators, in Eqs. (2) and (3), are assumed uncoupled to each other; the satellite oscillators are coupled to the master oscillator only. The summation in Eq. (2) is over a set of oscillators, where, again, (R) is the number of satellite oscillators in the set. From Eqs. (2) and (3) one obtains

$$Z_O(\omega)V_O(\omega) = P_e(\omega) \quad ; \quad V_r(\omega) = B(y,x_r,\eta_r)V_O(\omega) \quad , \tag{4}$$

where

$$Z_O(\omega) = (i\omega M)\{1-(y)^{-2}[1-s(y)]-i(y)^{-2}[\eta_O+\eta_s(y)]\} \quad , \tag{5}$$

$$[s(y)-i\eta_s(y)] = (y)^2 \sum_1^R A(y,\overline{m}_r,x_r,\eta_r)$$

$$A(y,\overline{m}_r,x_r,\eta_r) = \overline{m}_r B(y,x_r,\eta_r) \quad , \tag{6}$$

$$\begin{aligned} B(y,x_r,\eta_r) = {} & (x_r)^2[(1+\eta_r^2)(x_r)^2-(y)^2-i\eta_r(y)^2] \\ & \{[(x_r)^2-(y)^2]^2+[\eta_r(x_r)^2]^2\}^{-1} \quad , \end{aligned} \tag{7}$$

$$y=(\omega/\omega_O) \;;\; \overline{m}_r = (m_r/M) \;;\; x_r = (\omega_r/\omega_O) \;;\; 1\le r\le R \quad . \tag{8}$$

The quantity (x_r) defines the resonance frequency distribution of the satellite oscillators. It is emphasized that in the present paper this distribution, and not the spatial distribution of the

satellite oscillators, is in reference. The satellite oscillators are attached on a single velocity platform; that platform constitutes the mass of the master oscillator. The velocity in the plane of the platform is uniform so that each attached sprung mass (satellite oscillator) perceives the same velocity at the point of attachment. Whether the spatial distribution of the sprung masses on the platform is made to coincide with the resonance frequency distribution is moot in this paper. [cf. Fig. 1.] In some publications variations on this theme are implied but are not always strictly defined. Thus, the problem of a priori assuming a spatially discrete distribution of satellite oscillators (sprung masses) versus a spatially continuous distribution was briefly discussed in Reference 8. In this reference the satellite oscillators all possess the same resonance frequency. In Reference 6, on the other hand, a single couple structure with multiple resonance frequencies is used.

It may be useful to exemplify much of the analytical processes. For this purpose, two distinct resonance frequency distributions for the satellite oscillators; i.e., (x_r) as a function of (r), are depicted in Fig. 2. In Figs. 2a and 2b the resonance frequency distributions are [5, 7]

$$(\omega_r / \omega_o) = x_r = [1+\{1-2\bar{r}\}\gamma(\bar{R})]^{-1/2} \quad ; \quad 1 \le r \le R \ , \tag{9a}$$

and

$$(\omega_r / \omega_o) = x_r = \bar{r}(1-\bar{r})^{-1} \quad ; \quad 1 \le r \le R \ , \tag{9b}$$

respectively, where

$$\bar{r} = r(R+1)^{-1} \ ; \ \bar{R} = R(R+1)^{-1} \ ; \ \gamma(\bar{R}) = (\gamma / 2\bar{R}) < (1/2) \ . \tag{10a}$$

In keeping with References 7 and 5, a normalized mass distributions for the satellite oscillators; i.e., $(\overline{m}_r)$ as a function of (r), are also introduced and displayed in Fig. 2. In Figs. 2c and 2d the normalized mass distributions are

$$\overline{m}_r = (M_S / M)(R)^{-1} \quad ; \quad (M_S / M) = \sum_1^R \overline{m}_r \quad , \tag{9c}$$

$$\overline{m}_r = (M_S / M)[2/\pi(R+1)][(1-\overline{r})^2 + (\overline{r})^2]^{-1} \quad ;$$

$$(M_S / M) = \sum_1^R \overline{m}_r \quad , \tag{9d}$$

respectively, where (M_S / M) is the mass ratio of the total mass of the satellite oscillators to that of the mass of the master oscillator. In this paper this mass ratio is set at one-tenth; $(M_S / M) = (10)^{-1}$.

The impedance $Z_O(\omega)$, stated in Eq. (5), comprises the self-impedance $(i\omega M)$ $[1-(y)^{-2}(1+i\eta_O)]$ of the master oscillator and the sum over the impedances contributed by the satellite oscillators to which the master oscillator is coupled [7]. Typically, the impedance $(i\omega M)\, A(y, \overline{m}_r, x_r, \eta_r)$ is that contributed by the $(r)th$ oscillator; it is a function of the normalized frequency $y = (\omega / \omega_O)$ and is a functional of the normalized mass distribution $\overline{m}_r = (m_r / M)$, the normalized resonance frequency $x_r = (\omega_r / \omega_O)$ and the loss factor (η_r) of the $(r)th$ satellite oscillator. As Eqs. (1a) and (8) state, the normalizing frequency (ω_O) is the resonance frequency of the master oscillator in isolation. Under the same cover, the response $V_r(\omega)$, of the mass (m_r) of the $(r)th$ satellite oscillator, is related to the response $V_O(\omega)$, of the mass (M) of the master oscillator, by a factor that is a function of the

normalized frequency $y = (\omega / \omega_o)$ and is a functional of two of the three parameters; namely, of (x_r) and (η_r). The relationship between $V_r(\omega)$ and $V_o(\omega)$ is stated in Eq. (4).

Clearly (r) is a discrete variable. Can an *artificially* assigned (r) set to be a continuous variable? This proposed continuity of (r) is exemplified by the solid-line curves in Fig. 2. It is convenient, in this connection, to normalize (scale) the continuous variable (r) by the number of satellite oscillators (R) *plus* unity (1), as suggested by Eqs. (9) and (10). The normalized (r) is designated $(\bar{r})$ and the dependent parameters are then expressed in the forms

$$\overline{m}_r \Rightarrow \overline{m}(\bar{r}) \;\; ; \;\; x_r \Rightarrow x(\bar{r}) \;\; ; \;\; \eta_r \Rightarrow \eta(\bar{r}) \quad . \tag{11}$$

The scaled forms of (x_r); namely, $x(\bar{r})$, in Eqs. (9a) and (9b), are depicted as functions of $(\bar{r})$ in Figs. 3a and 3b, respectively. In this vein, Eqs. (4)-(7) may be recast, without further ado, in the forms

$$Z_o(\omega)V_o(\omega) \;=\; P_e(\omega) \;\; ; \;\; V(\bar{r},\omega) \;=\; B\{y,\; x(\bar{r}),\; \eta(\bar{r})\}V_o(\omega) \quad , \tag{12}$$

where

$$Z_o(\omega) \;=\; (i\omega M)\{1-(y)^{-2}[1-s(y)]-i(y)^{-2}[\eta_o+\eta_s(y)]\} \tag{13}$$

$$[s(y)-i\eta_s(y)] \;=\; (y)^2\int_{(\varepsilon)}^{(R+\varepsilon)} dr\;\; A\{y,\; \overline{m}(\bar{r}),\; x(\bar{r}),\; \overline{\eta}(\bar{r})\} \;\; ;$$

$$A\{y,\; \overline{m}(\bar{r}),\; x(\bar{r}),\; \eta(\bar{r})\} \;=\; \overline{m}(\bar{r})\;\; B\{y,\; x(\bar{r}),\; \eta(\bar{r})\} \quad , \tag{14a}$$

$$[s(y)-i\eta_S(y)] = (y)^2 \int_{(\bar{\varepsilon})}^{(\bar{R}+\bar{\varepsilon})} d\bar{r}\ A\{y,\ \mu(\bar{r}),\ x(\bar{r}),\ \eta(\bar{r})\} \ ;$$

$$A\{y,\ \mu(\bar{r}),\ x(\bar{r}),\ \eta(\bar{r})\} = \mu(\bar{r})\ B\{y,\ x(\bar{r}),\ \eta(\bar{r})\} \ , \qquad (14b)$$

$$B\{y,\ x(\bar{r}),\ \eta(\bar{r})\} = \{x(\bar{r})\}^2\ [(1+\{\eta(\bar{r})\}^2)\ \{x(\bar{r})\}^2-(y)^2-i\eta(\bar{r})(y)^2]$$

$$\{[\{x(\bar{r})\}^2-(y)^2]^2 + [\eta(\bar{r})\{x(\bar{r})\}^2]^2\}^{-1} \ , \qquad (15)$$

$$\overline{m}_r \Rightarrow \mu(\bar{r})\ (R+1)^{-1} \ ,\ \bar{\varepsilon} = \varepsilon(R+1)^{-1}\ \ \bar{R} = R(R+1)^{-1} \ . \qquad (10b)$$

The parameter $(\bar{\varepsilon})$ designates the end-effects that are introduced in the transition from the discrete to the continuous domains; e.g., $\bar{\varepsilon} = \varepsilon(R+1)^{-1}$, with $\varepsilon \approx 1/2$, say. [cf. Fig. 2.]

Afresh, consider the mechanical system comprising of a master oscillator, with mass (M) and stiffness (K), that is coupled to a set of satellite oscillators, as defined in Eq. (1) and sketched in Fig. 1. However, now the resonance frequencies $\{\omega(\zeta)\}$ of the satellite oscillators are assumed to be a priori continuously, but not necessarily uniformly, distributed in the interval $0<\zeta<1$ [5]. Example of two typical resonance frequency distributions; i.e., $[x(\zeta)]$ as functions of (ζ), are depicted in Fig. 3. In Figs. 3a and 3b the normalized resonance frequency distributions are

$$[\omega(\zeta)/\omega_O] = x(\zeta) = [1+(1-2\zeta)\gamma(\bar{R})]^{-(1/2)} \ ;\ (\bar{R})<(1/2)\ ;\ 0<\zeta<1 \ , \qquad (16a)$$

$$[\omega(\zeta)/\omega_O] = x(\zeta) = (\zeta)(1-\zeta)^{-1} \ ;\ \ 0<\zeta<1 \ , \qquad (16b)$$

respectively, and $\gamma(\overline{R})$ is defined in Eq. (10) [5, 7]. [cf. Eqs. (9a) and (9b), respectively, and Eq. (10)]. The corresponding normalized mass distributions $\mu(\zeta)$ of the satellite oscillators are [5 7,]

$$\mu(\zeta) = (M_S / M)(\overline{R})^{-1} \;\; ; \;\; 0 < \zeta < 1 \;\; , \tag{16c}$$

$$\mu(\zeta) = (M_S / M)(2/\pi) \, [(1-\zeta)^2 + (\zeta)^2]^{-1} \;\; ; \;\; 0 < \zeta < 1 \;\; , \tag{16d}$$

[cf. Eqs. (9c) and (9d), respectively, and Eq. (10).] The linear equations of motion of the master oscillator insitu and of a typical satellite oscillator insitu are

$$Z_O(\omega) \; V_O(\omega) \; = \; P_e(\omega) \;\; ; \;\; V(\zeta,\omega) \; = \; B\{y, \; x(\zeta), \; \eta(\zeta)\} \; V_O(\omega) \;\; , \tag{17}$$

where

$$Z_O(\omega) \; = \; (i\omega M)\{1-(y)^{-2}[1-s^C(y)] - i(y)^{-2}[\eta_O + \eta_S^C(y)]\} \;\; , \tag{18}$$

$$[s^C(y) - i\eta_S^C(y)] = (y)^2 \int_0^1 d\zeta \;\; A\{y, \; \mu(\zeta), \; x(\zeta), \; \eta(\zeta)\} \;\; ;$$

$$A\{(y, \; \mu(\zeta), \; x(\zeta), \; \eta(\zeta)\} \; = \; \mu(\zeta) B\{y, \; x(\zeta), \; \eta(\zeta)\} \;\; , \tag{19}$$

$$B\{y,\ x(\zeta),\ \eta(\zeta)\} = \{x(\zeta)\}^2\ [\{1+\ \eta^2(\zeta)\}\{x(\zeta)\}^2 - (y)^2\ - i\eta(\zeta)(y)^2]$$

$$\{[\{x(\zeta)\}^2\ -\ (y)^2]^2 + [\eta(\zeta)\{x(\zeta)\}^2]^2\}^{-1}\ , \tag{20}$$

$$y = (\omega / \omega_o)\quad ;\quad x(\zeta)\ =\ [\omega(\zeta) / \omega_o]\quad ;\quad 0 < \zeta < 1\quad . \tag{21}$$

A question arises: Under what conditions can the linear Eqs. (4)-(7) be asymptotically circumvented into the linear Eqs. (12)-(15) and Eqs. (12)-(15) into the linear Eqs. (17)–(20), respectively, and are these conditions significant? This question has been partially answered in previous publications; in this paper, a more definitive answer is sought [2,7]. However, prior to this pursuit, one may inquire: Suppose these asymptotic circumventions were found permissible as these are stated, can one perform, a priori, the integration in the continuous domain that, in fact, replaces the summation in the discrete domain?

II. Evaluation of the Induced Loss Factor $[\eta_s(y)]$ in Eqs. (13)-(15)

The following quantities are defined first

$$z(\overline{r}) = [x(\overline{r}) / y] \;;\; [d\{x(\overline{r})\} / d\overline{r}]^{-1} = f(\overline{r}) \quad . \tag{22}$$

Then, from Eqs. (13) – (15), one obtains

$$\eta_S(y) = (y)^3 \int_{z(\overline{\varepsilon})}^{z(\overline{R}+\overline{\varepsilon})} dz(\overline{r})[f(\overline{r})\mu(\overline{r})][\{z(\overline{r})\}^2\eta(\overline{r})] \;.$$

$$\{[\{z(\overline{r})\}^2 - 1]^2 + [\{z(\overline{r})\}^2\eta(\overline{r})]^2\}^{-1} \quad . \tag{23}$$

Were the factor $[f(\overline{r})\mu(\overline{r})]$ to be a well behaved function of $(\overline{r})$ in the range $\overline{\varepsilon} < \overline{r} < \overline{R} + \overline{\varepsilon}$ and were $\eta(\overline{r})$ to be small compare with unity, as a function of $(\overline{r})$ in that same range, the integral in Eq. (23) immediately yields the result

$$\eta_S(y) = (\pi / 2)(y)^3[f(\overline{r}_O)\mu(\overline{r}_O)] \;;\; z(\overline{r}_O) = 1 \;;\; (\overline{\varepsilon}) < \overline{r}_O < (\overline{R} + \overline{\varepsilon}) \quad , \tag{24}$$

which, clearly, is independent of $\eta(\overline{r})$ provided these loss factors are set small compared with unity in the range $(\overline{\varepsilon}) < \overline{r} < \overline{R} + \overline{\varepsilon}$ [2]. [The *well behaved* function $[f(\overline{r})\mu(\overline{r})]$ of $(\overline{r})$ and the smallness of the loss factor $\eta(\overline{r})$ as compared with unity, both in the range $(\overline{\varepsilon}) < \overline{r} < (\overline{R} + \overline{\varepsilon})$ are specified as such in order to ensure that the contribution to the integral by the resonance peaks of the integral at $z(r) \simeq 1$ are dominant. Were contributions by the peaks, in certain ranges of (y), not to be dominant, the integral, as just performed will naturally underestimate the value of the integral in these ranges of (y). In the present paper, situations that may fall into this category are neither evaluated nor specifically considered.

Indeed, this is the reason that only the induced damping is evaluated and considered in this paper [8].] Employing the normalized resonance frequency distributions stated in Eqs. (9a) and (9b), one finds from Eqs. (22) and (24) that

$$f(\bar{r}_O) = [\gamma(\bar{R})(y)^3]^{-1} \ ; \ [1+(\gamma/2)]^{-(1/2)} < y < [1-(\gamma/2)]^{-(1/2)} \ , \tag{25a}$$

$$f(\bar{r}_O) = (1+y)^{-2} \ ; \ (2R+1)^{-1} < y < (2R+1) \ , \tag{25b}$$

respectively, where (γ) is defined in Eq. (10) and is chosen, herein, to be equal to (0.6). Similarly, employing the normalized mass distributions stated in Eqs. (9c) and (9d), one finds from Eqs. (22) and (23) that

$$\mu(\bar{r}_O) \Rightarrow (M_S / M)(\bar{R})^{-1} \ ; \ (\bar{\varepsilon}) < \bar{r}_O < (\bar{R}+\bar{\varepsilon}) \ , \tag{25c}$$

$$\mu(\bar{r}_O) \Rightarrow (M_S / M)(2/\pi) \ (1+y)^2 \ [1+(y)^2]^{-1} \ ; (\bar{\varepsilon}) < \bar{r}_O < (\bar{R}+\bar{\varepsilon}) \ . \tag{25d}$$

From Eqs. (24) and (25) one readily derives, for the two examples herein considered, the results

$$\eta_S(y) = (M_S / M)[\pi / \{2\gamma(\bar{R})\}] \ ; [1+(\gamma/2)]^{-(1/2)} < \ y < \ [1-(\gamma/2)]^{-(1/2)} \ , \tag{26a}$$

$$\eta_S(y) = (M_S / M)(y)^3[1+(y)^2]^{-1} \ ; \ [(2R+1)]^{-1} < y < [(2R+1)] \ . \tag{26b}$$

It is recognized that $\eta_S(y)$ is largely meaningful only at and in the vicinity of (y) equal to unity; $y \approx 1$, so that the restricted range of (y) in Eq. (26) is not physically significant. For

the two examples, stated in Eqs. (26a) and (26b), $\eta_S(y)$ is displayed in Figs. 4a and 4b, respectively. The dashed-line curves in these figures are merely free extrapolations beyond the permissible ranges.

Since it emerges that $\eta_S(y)$ is independent of the loss factors $\eta(\bar{r})$, provided $\eta(\bar{r}) << 1$ in the range $(\bar{\varepsilon}) < \bar{r} < (\bar{R} + \bar{\varepsilon})$, these loss factors of the satellite oscillators may, indeed, be set equal to zero and nobody would be the wiser. Is that true?

Finally, it is remarked that the difference between the induced loss factor $\eta_S(y)$, stated in Eq. (14), and $\eta_S^C(y)$ stated in Eq. (19), is the number (R) of satellite oscillators. Clearly, Eq. (19) can be derived from Eq. (14) by allowing (R) to increase enough to render $(\bar{R} + \bar{\varepsilon}) \rightarrow 1$ and $(\bar{\varepsilon}) \rightarrow 0$.

III. Modal Overlap Condition for Relating the Discrete and the Continuous Domains

To establish the relationship between the discrete and the continuous domains one may define two distinct frequency bands. The first, designated $(\delta\omega_r)$ defines the fair frequency territory occupied by each satellite oscillator with respect to its adjacent neighbors; this frequency bandwidth is centered on the resonance frequency of that satellite oscillator. The expression for this bandwidth is

$$\delta\omega_r = \{(1/2)(\omega_{r+1} - \omega_{r-1})\ [U(R-1+\varepsilon-r) - U(2-\varepsilon-r)]$$

$$+ (\omega_2 - \omega_1)U(1+\varepsilon-r) + (\omega_R - \omega_{R-1})U(r+\varepsilon-R)\}\, U(r-\varepsilon)U(R+\varepsilon-r) \quad , \ (27a)$$

where $U(\alpha)$ is the usual unit step function and (ε) is less than unity, equal to $(1/2)$, again, say. The inverse of the frequency bandwidth $(\delta\omega_r)$ is commonly referred to as the *modal density* (n_r) of the satellite oscillators; i.e.,

$$n_r = (\delta\omega_r)^{-1} \quad ; \quad \varepsilon \le r \le R+\varepsilon \quad , \qquad (28)$$

where the inversion of $(\delta\omega_r)$ is carefully executed with respect to the unit step functions [10, 11]. The second frequency band, designated $(\Delta\omega_r)$, defines the inherent frequency bandwidth of the $(r)th$ satellite oscillator; this bandwidth accounts for the inherent damping of this satellite oscillator. The measure of the damping, in turn, defines the loss factor (η_r). The bandwidth $(\Delta\omega_r)$ is then stated in terms of the loss factor (η_r) and the resonance frequency (ω_r) in the form

$$\Delta\omega_r = (\omega_r \eta_r) U(r-\varepsilon) U(R+\varepsilon-r) \quad . \tag{27b}$$

The ratio (b_r) of these two frequency bandwidths defines a *modal overlap parameter*

$$\{b_r = (\Delta\omega_r / \delta\omega_r) = (n_r \omega_r \eta_r)\} U(r-\varepsilon) U(R+\varepsilon-r) \quad , \tag{29}$$

where use is made of Eqs. (27) and (28).

For a modal overlap parameter (b_r) that is less than unity; $b_r < 1$, adjacent satellite oscillators reside outside each other's bandwidths. Consequently, the influence of the satellite oscillators on the response of the master oscillator, as a function of (y); $y = (\omega / \omega_o)$, can be identified individually; each contribution associated with a satellite oscillator stand out prominently from the others. An example of such an influence in terms of evaluating the induced loss factor $\eta_s(y)$, as a function of (y) with (b_r) less than unity, is depicted in Fig. 5 [5, 7]. On the other hand, for a modal overlap parameter (b_r) that exceeds the value of unity; $b_r > 1$, adjacent satellite oscillators reside within each other's bandwidths. Therefore, their influence on the response of the master oscillator is largely continuous as a function of (y); $y = (\omega / \omega_o)$. The more the modal overlap parameter (b_r) exceeds the value of unity, the more the continuity [7]. An example of such an influence, in terms of evaluating the induced loss factor $\eta_s(y)$, as a function of (y) with (b_r) exceeding unity, is depicted in Fig. 6 [5, 7]. Comparing Fig. 4 with Fig. 5 and with Fig. 6, respectively, serves to expose the issue that is central to this paper. Figures 4a and 4b, respectively, overlap prime portions of Figs. 6a and 6b only. These prime portions are defined within the range set on (y) in Eqs. (26a) and (26b), respectively. No such overlap exists between Figs. 4a and 4b and

Figs. 5a and 5b, respectively. Nonetheless, if the undulations in Fig. 5 are appropriately averaged, the overlap between Fig. 4 and 6 is extended to include the mean values of Fig. 5 [7, 12]. [cf. Appendix A.] When (b_r) becomes small compared with unity the mean values alone do not reflect the presence of undulations and, therefore, Fig. 4 cannot serve to substitute for Fig. 5. The information that lies in Fig. 5 can hardly be derived from Fig. 4; Fig. 4 is akin to Fig. 6 but not to Fig. 5. Since Fig. 4 is derived by replacing the summation by integration, it is concluded that this replacement is commensurate with the imposition of modal overlap parameters that exceed unity; $b_r > 1$. [cf. Appendix A.] An imposition of this kind ensures that the evaluations of mean-value data do not conceal information that may render the data, at best, ambiguous and, at worst, misleading [8, 13].

When the modal overlap parameter (b_r) exceeds unity, the *modal overlap condition* is said to be satisfied. When the modal overlap condition is satisfied, the linear dynamic description $[s(y) - i\eta_s(y)]$, comprising the sum of the individual linear dynamic descriptions of all the satellite oscillators; e.g., typically the term $[(y)^2 A(y, \overline{m}_r, x_r, \eta_r)]$ contributed to the sum by the $(r)th$ satellite oscillator, may be, well nigh, evaluated by integration

$$[s(y) - i\eta_s(y)] \;=\; (y)^2 \sum_{1}^{R} A(y, \overline{m}_r, x_r, \eta_r)$$

$$\Rightarrow (y)^2 \int_{(\varepsilon)}^{(R+\varepsilon)} dr\, A\{y, \overline{m}(r),\, x(r), \eta(r)\} \quad , \tag{30a}$$

where (r), in the second of Eq. (30a), is a continuous and dimensionless variable [2]. [cf. Eqs. (6) and (14a).] It is emphasized that the transition, described in Eq. (30a), is validated if, and only if, it is implicitly understood that (b_r), as stated in Eq. (29), exceeds unity; namely

$$\{b_r = (n_r \omega_r \eta_r) > 1\} \; U(r-\varepsilon) \; U(R+\varepsilon-r) \quad . \tag{31a}$$

The assignment of continuity to (r), in Eq. (30a), renders the normalized resonance frequency $x_r \Rightarrow x(r)$ a continuous function of (r), as exemplified in Figs. 2a and 2b [14]. Similarly, $\overline{m}_r \Rightarrow \overline{m}(r)$ and $\eta_r \Rightarrow \eta(r)$. [cf. Figs. 2c and 2d.] In this rendering, Eqs. (27a) and (27b) may be approximated, by smoothing, in the forms

$$n(r) = [\delta\omega(r)]^{-1} \simeq [d\omega(r)/dr]^{-1} \; U(r-\varepsilon) \; U(R+\varepsilon-r) \quad , \tag{32a}$$

$$\Delta\omega(r) = \omega(r)\eta(r) \quad ; \quad (\varepsilon) < r < (R+\varepsilon) \quad , \tag{32b}$$

respectively, where one recognizes that $[d\omega(r)/d(r)]$ is, by definition, a positive quantity and Eq. (28) is referenced. From Eq. (22) one finds that $[d\omega(r)/d(r)] \equiv [(R+1)f(\overline{r})/\omega_o]^{-1}$. Under the guidance of Eq. (32a) and the normalization (scaling) proposed in Eq. (10), Eq. (9) yields the two corresponding modal densities

$$n(r) = n(\overline{r}) = [(R+1)/\omega_o\gamma(\overline{R})]\{x(\overline{r})\}^{-3} \; U(\overline{r}-\overline{\varepsilon}) \; U(\overline{R}+\overline{\varepsilon}-\overline{r}) \quad , \tag{33a}$$

$$n(r) = n(\overline{r}) = [(R+1)/\omega_o](1-\overline{r})^2 \; U(\overline{r}-\overline{\varepsilon}) \; U(\overline{R}+\overline{\varepsilon}-\overline{r}) \quad , \tag{33b}$$

respectively, where, again, $\overline{\varepsilon} = \varepsilon(R+1)^{-1}$, $\overline{R} = R(R+1)^{-1}$, $\overline{r} = r(R+1)^{-1}$ and $\gamma(R) = (\gamma/2\overline{R})$. From Eq. (31a), the modal overlap condition is satisfied, in the continuous domain, provided

$$\{b(r)=b(\bar{r})=n(r)\,\omega(r)\,\eta(r)=n(\bar{r})\,\omega(\bar{r})\,\eta(\bar{r})>1\}\ U(\bar{r}-\bar{\varepsilon})\ U(\bar{R}+\bar{\varepsilon}-\bar{r})\ . \tag{31b}$$

In terms of Eqs. (33a) and (33b), the specific modal overlap conditions, per Eq. (31b), are

$$\{b(r)=b(\bar{r})=[x(\bar{r})]^{-2}[(R+1)\eta(\bar{r})/\gamma(\bar{R})]>1\ \}\ U(\bar{r}-\bar{\varepsilon})\ U(\bar{R}+\bar{\varepsilon}-\bar{r})\ , \tag{34a}$$

$$\{b(r)=b(\bar{r})=[\bar{r}(1-\bar{r})][(R+1)\eta(\bar{r})]>1\}\ U(\bar{r}-\bar{\varepsilon})\ U(\bar{R}+\bar{\varepsilon}-\bar{r})\ , \tag{34b}$$

respectively. It is emphasized that the transition, from the discrete to the continuous domains, state in Eq. (30a), is predicated on the validity of Eq. (31), in general, and on Eq. (34) specifically for the two examples here pursued. To facilitate the assessment of the transition, from the discrete to the continuous domains, it is convenient to scale Eq. (30a). The continuous (r) is, in that scaling, normalized by the number $(R+1)$, as already performed for example in Eq. (33). The so normalized continuous (r) is designated (ζ) and hence, in that vein, Eqs. (31)–(34) are to be recast with $\bar{r}\to(\zeta)$; e.g., in this normalization Eq. (32) is written in the form

$$\{n(\zeta)d\omega(\zeta)\ =\ (R+1)d\zeta\}\ U(\zeta-\bar{\varepsilon})\ U\{\bar{R}+\bar{\varepsilon}-\zeta\}\ , \tag{35a}$$

$$\Delta\omega(\zeta)\ =\ \omega(\zeta)\eta(\zeta)\ \ ,\ (\bar{\varepsilon})\ <\ \zeta\ <\ (\bar{R}+\bar{\varepsilon})\ \ , \tag{35b}$$

respectively, where

$$\zeta = r\ (R+1)^{-1} \quad ; \quad \bar{\varepsilon} = \varepsilon\ (R+1)^{-1} \ ; \ \bar{R} = R\ (R+1)^{-1} \quad . \tag{36a}$$

The scaling is illustrated in Fig. 3 for the examples depicted in Figs. 2a and 2b. [cf. Eqs. (9) and (16).] Employing this scaling in Eq. (30a), one obtains

$$[s(y) - i\eta_S(y)] = (y)^2 \int_{(\varepsilon)}^{(R+\varepsilon)} dr\ A\{y, \overline{m}(r), x(r), \eta(r)\}$$

$$\Rightarrow (y)^2 \int_{(\bar{\varepsilon})}^{(\bar{R}+\bar{\varepsilon})} d\zeta\ A\{y, \mu(\zeta), x(\zeta), \eta(\zeta)\} \ , \tag{30b}$$

where (ζ) and $(\bar{\varepsilon})$ are defined in Eq. (36a). [cf. Eqs. (10) and (14).] If one further assumes that the number (R) of the satellite oscillators is large compared with unity, Eq. (30b) becomes

$$[s(y) - i\eta_S(y)] \Rightarrow (y)^2 \int_0^1 d\zeta\ A\{y, \mu(\zeta), x(\zeta), \eta(\zeta)\}$$

$$= [s^C(y) - i\eta_S^C(y)] \quad , \tag{30c}$$

where $[s^C(y) - i\eta_S^C(y)]$ is defined in Eq. (19) and it follows from Eq. (36a) that

$$\bar{\varepsilon} \to o \quad ; \quad (\bar{R}+\bar{\varepsilon}) \to 1 \quad ; \quad \bar{R} \to 1 \text{ as } R >>> 1 \quad . \tag{36b}$$

The modal overlap condition that validates Eqs. (30b) and (30c) is derived, from Eq. (31b), in the form

$$\{b(\zeta) \;=\; \omega(\zeta)\;[d\omega(\zeta)/d\zeta]^{-1}[(R+1)\eta(\zeta)]>1\;\}\;U(\zeta-\bar{\varepsilon})\;U(\bar{R}+\bar{\varepsilon}-\zeta) \quad , \tag{31c}$$

where use is made of Eq. (35a). For the examples quoted in Eq. (9), Eq. (31c) assumes the forms

$$\{b(\zeta) \;=\; [x(\zeta)]^{-2}\;[(R+1)\eta(\zeta)/\gamma(\bar{R})]>1\}\;U(\zeta-\bar{\varepsilon})\;U(\bar{R}+\bar{\varepsilon}-\zeta) \quad , \tag{37a}$$

$$\{b(\zeta) \;=\; [\zeta(1-\zeta)]\;[(R+1)\eta(\zeta)]>1\}\;U(\zeta-\bar{\varepsilon})\;U(\bar{R}+\bar{\varepsilon}-\zeta) \quad , \tag{37b}$$

where in Eq. (37a)

$$[x(\zeta)]^{-2} \;=\; [1+(1-2\zeta)\gamma(\bar{R})] \quad ; \quad \gamma(\bar{R}) \;=\; (\gamma/2\bar{R})<(1/2) \quad . \tag{38}$$

[cf. Eq. (16a).] It emerges, therefore, that the validity of Eqs. (30b) and (30c) demands that Eq. (31c), in general, and Eqs. (37a), and (37b) in particular, are obeyed. These latter three equations may be cast in the form

$$\{[(R+1)\;\eta(\zeta)]>\begin{Bmatrix}[d\ln\{\omega(\zeta)\}/d\zeta] \\ [\gamma(\bar{R})\{x(\zeta)\}^{2}] \\ [\zeta(1-\zeta)]^{-1}\end{Bmatrix}\}\;U(\zeta-\bar{\varepsilon})\;U(\bar{R}+\bar{\varepsilon}-\zeta) \quad \begin{matrix} , & (31\text{d}) \\ , & (37\text{c}) \\ . & (37\text{d})\end{matrix}$$

The quantity $[d\ln\{\omega(\zeta)\}/d\zeta] \equiv [d\ln\{x(\zeta)\}/d\zeta]$, in Eq. (31d), is the local slope in figures of which Figs. 3a and 3b serve as two specific examples; i.e., the examples given in Eqs. (37c) and (37d), respectively [14]. [It is noted that Eq. (31d) may be cast in the alternate form

$$\eta(r) \; > \; [d\ln\{\omega(r)\}/dr] \equiv [d\ln\{x(r)\}/dr] \quad . \tag{31e}$$

In Eq. (31e) the local slope is that of curves like those exemplified in Figs. 2a and 2b for the continuous (r) [14]. Again, it is emphasized that the relationship between Figs. 2a and 2b and Figs. 3a and 3b, respectively, is merely the scale factor $(R+1)$. In that sense Eq. (31d) is merely the normalized version of Eq. (31e) and, as such, the former is to be preferred when comparing situations of varying numbers of satellite oscillators [5].] Equations (37c) and (37d) are depicted graphically in Figs. 7a and 7b, respectively. These equations and figures exemplify that $[\eta(\zeta)]$ cannot be independently set equal to zero without qualification. The lower limit that can be assigned to $[\eta(\zeta)]$ is dependent on the number (R) of satellite oscillators plus unity (1). The value of $[(R+1)\eta(\zeta)]$ may be large or small compared with unity, but it must remain finite if Eqs. (30b) and (30c) are to be validated. The only way to allow for infinitesimal $[\eta(\zeta)]$'s; i.e., nearly undamped satellite oscillators, is to explicitly or implicitly render $(R+1)^{-1}$ proportionately infinitesimal. In particular, the number of satellite oscillators needs to approach infinity at least at the same rate that $[\eta(\zeta)]$ approaches infinitesimal values. Thus, once the modal overlap condition is satisfied, the satellite oscillators collectively possess enough damping to account for the physical presence of the induced loss factor.

Acknowledgement

The author is, and some readers will be, thankful to K. J. Becker for making it possible to present some of the equations and ideas in graphical forms.

THIS PAGE INTENTIONALLY LEFT BLANK

Appendix A: A Few Additional Remarks

In a steady state analysis of the dynamic characteristics of the response of structural complexes, many data sets are presented in terms of appropriately averaged values. Such presentations are ultimately acceptable as long as these mean values are all the information that is either available, sought or both. The loss factor $\eta_S(y)$ presented in Fig. 6, which pertains to a modal overlap parameter (b_r) that is equal to two; $b_r = 2$, simultaneously fits the two categories. On the other hand, the loss factor $\eta_S(y)$ depicted in Fig. 5, which pertains to a modal overlap parameter (b_r) that is equal to one tenth; $b_r = 10^{-1}$, does not meet either categories. Figure 4, however, cast the data in Fig. 5 in terms of mean values, thus meeting the availability category only; the data sought may demand some form of accounting for the undulations. The undulations are excursions in the values of $\eta_S(y)$, as a function of the normalized frequency (y), between resonance peaks and anti-resonance nadirs. Since the size of the excursions are determined by the modal overlap parameter (b_r), can a measure of the undulations be devised as a function of this parameter? The excursions, it is noted, are the larger the smaller are the modal overlap parameters compared with unity. One is reminded in this connection, however, that to a first order of approximation results obtained from averaging the undulations; e.g., the appropriately averaged values in Fig. 5, as well as the results obtained by performing the integration in Eq. (23); e.g., the data depicted in Fig. 4, are found to be independent of the individual loss factors of the satellite oscillators [1-7]. Therefore, to that approximation the mean values also lack dependence on the modal overlap parameter (b_r); e.g., although there is no demand for equal values of this parameter, there is an overlap between Figs. 4 and 6. On the other hand, although the values of the modal overlap parameters may be identical in Figs. 4 and 5, there is no overlap between prime regions in

these figures, except in a qualified sense. Indeed, when mean values only are to be reported, to avoid "a tail wagging a dog," it is to be assumed that the modal overlap parameters are set in excess of unity, unless they are otherwise specifically stated [13]. Again, when values are derived by appropriately averaging undulating data, the value of the modal overlap parameter (b_r) must accompany the specification of these data; e.g., when Fig. 4 is meant to represent the mean values of Fig. 5, the modal overlap parameter (b_r) must be stated as one tenth; $b_r = 10^{-1}$. Otherwise, Fig. 4 is meant to represent Fig. 6 (for which $b_r = 2$) and not, at all, Fig. 5.

In this connection, it transpires that the first order approximation just discussed is the better the larger is the number (R) of the satellite oscillators and the closer is the modal overlap parameter (b_r) to unity. Indeed, for low values of (R) and for large values of (b_r), erosions may beset the exact data obtained via the summation in Eq. (6) [7]. Such simultaneous decreases in the number of satellite oscillators and increases in the modal overlap parameters may not only render the exact data dependent on these quantities, but may also violate the requirement that the contributions to the integral be dominated by the resonances in the integrand. [cf. Eqs. (23) and (24).] Reference 7 suggests that to ensure a first order approximation that is free of erosions one must require that $\overline{b}_r = [b_r(R+1)^{-1}] << 1$. On the other hand, Eqs. (31d), (37c) and (37d) suggest that to ensure that mean values are the prime data, (b_r) must exceed unity; $b_r > 1$.

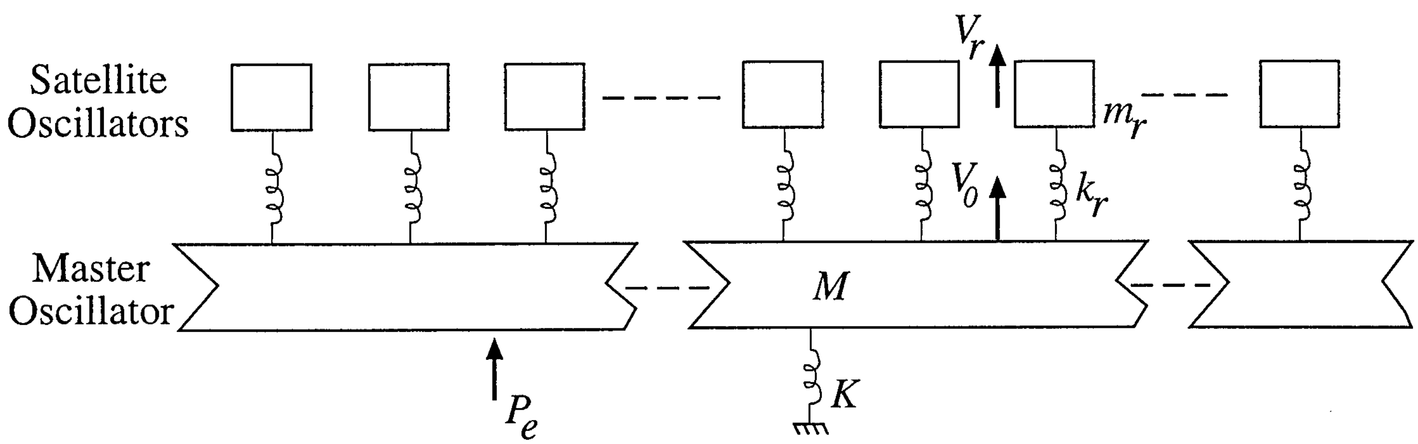


Fig. 1. Master oscillator coupled to a set of resonance frequency distributed satellite oscillators. The master oscillator is defined by the mass (M) and the stiffness (K) and the $(r)th$ satellite oscillator by mass $(m_r)$and stiffness (k_r). Only the master oscillator is driven, by an external drive $P_e(\omega)$, generating the response $V_o(\omega)$ in the master oscillator and the response $V_r(\omega)$ in the $(r)th$ satellite oscillator.

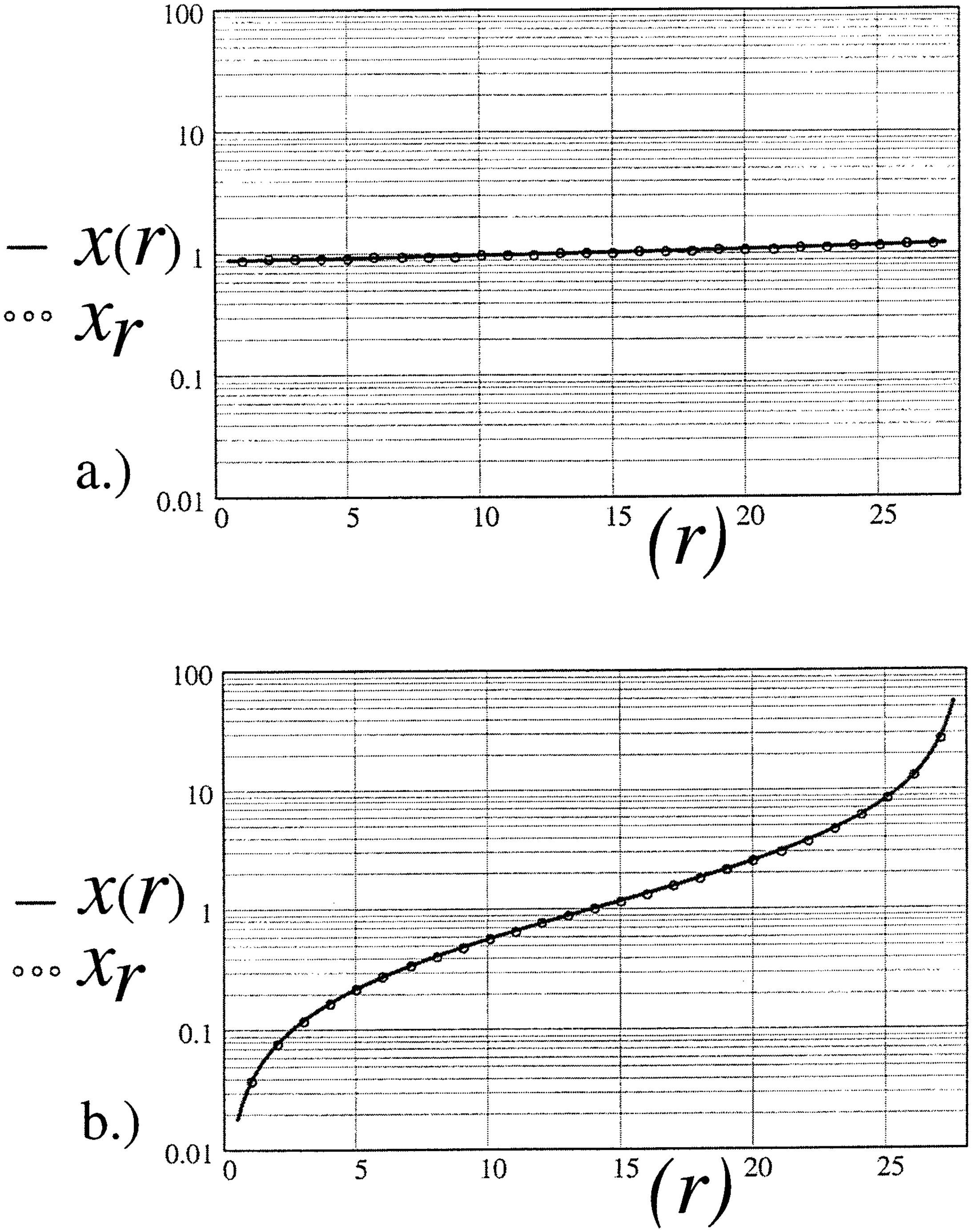
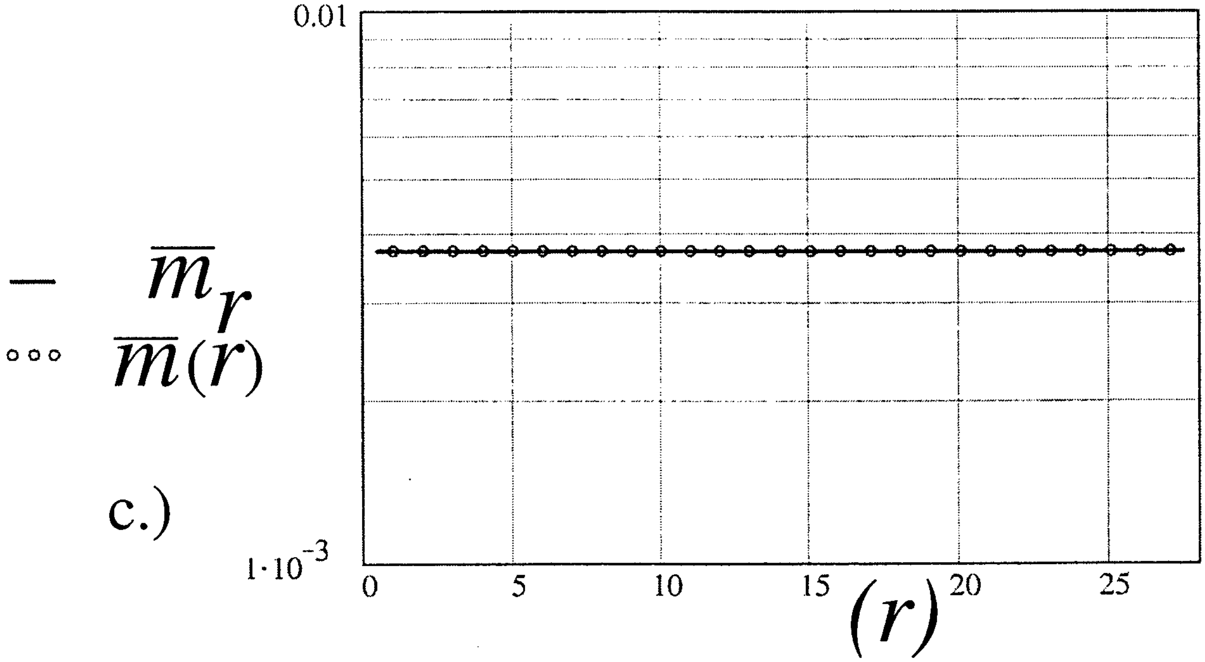


Fig. 2. Resonance frequency distribution; (x_r), and normalized mass distribution; $(\overline{m}_r)$, as functions of the discrete (r) (distinct dots), and $x(r)$ and $\overline{m}(r)$, as functions of the continuous (r) (solid line curves). Number (R) of satellite oscillators is twenty seven; $(R = 27)$ and mass ratio (M_S / M) is a tenth (10^{-1}).

a. (x_r) as specified in Eq. (9a). b. (x_r) as specified in Eq. (9b).



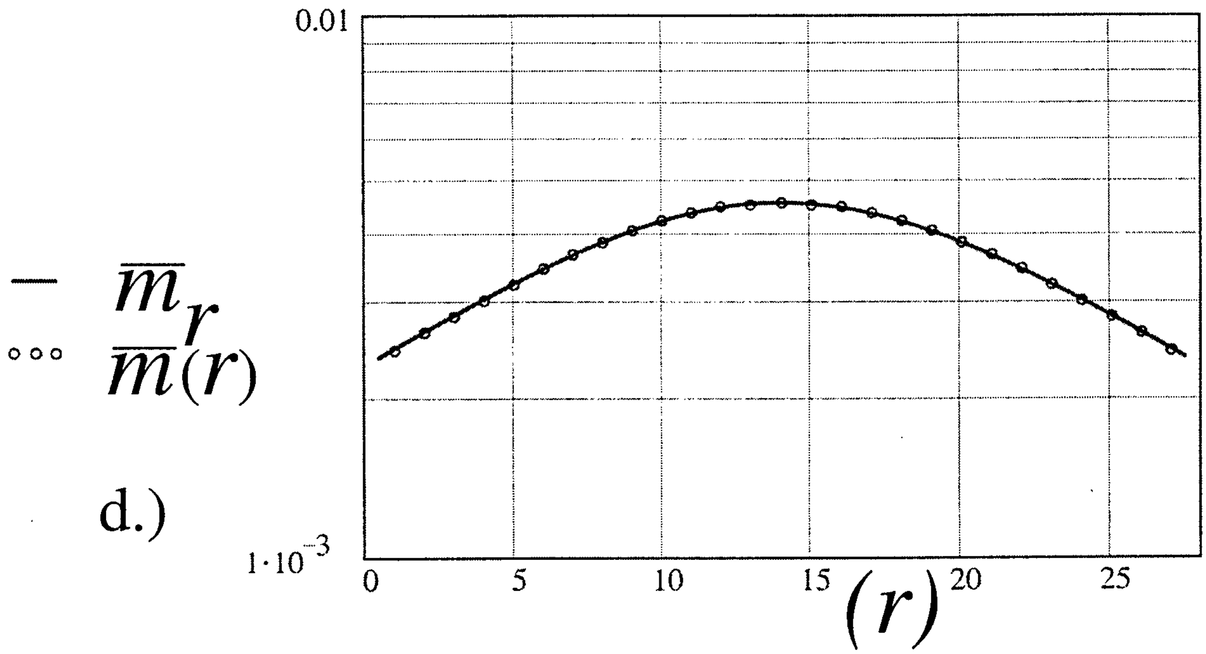


Fig. 2. Resonance frequency distribution; (x_r), and normalized mass distribution; $(\overline{m}_r)$, as functions of the discrete (r) (distinct dots), and $x(r)$ and $\overline{m}(r)$, as functions of the continuous (r) (solid line curves). Number (R) of satellite oscillators is twenty seven; $(R = 27)$ and mass ratio $(M_S \,/\, M)$ is a tenth (10^{-1}).

c. $(\overline{m}_r)$ as specified in Eq. (9c). d. $(\overline{m}_r)$ as specified in Eq. (9d).

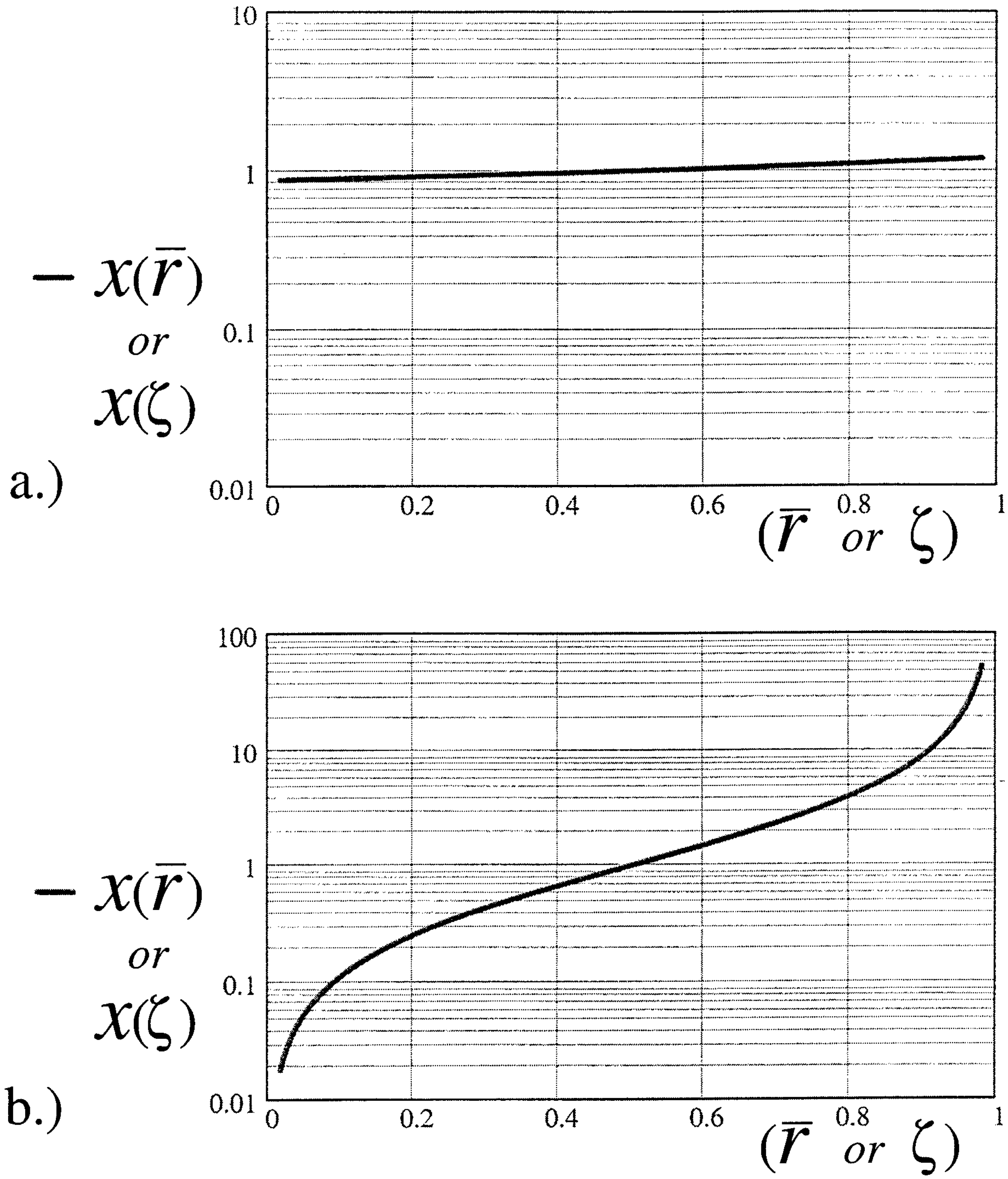
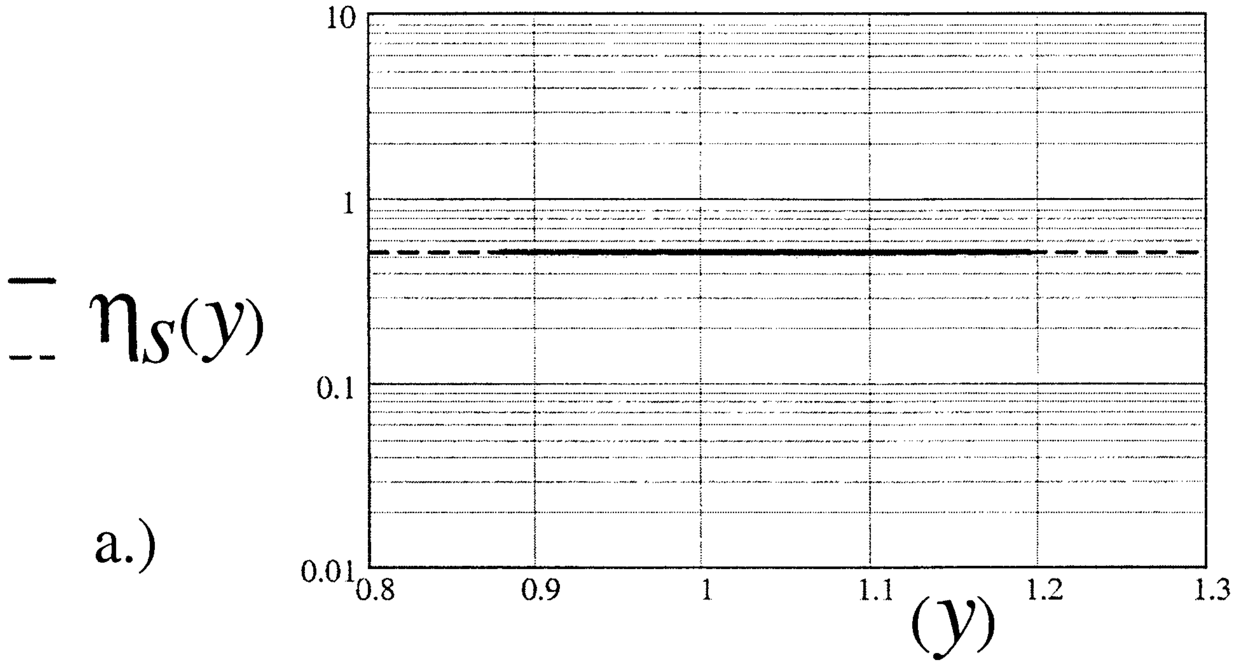


Fig. 3. Resonance frequency distribution; $\chi(\bar{r})$ or $\chi(\zeta)$ as a function of the normalized continuous variable $(\bar{r})$ or (ζ), respectively. [cf. Fig. 2.] $(R = 27)$.

a. $\chi(\zeta)$ as specified in Eq. (16a). b. $\chi(\zeta)$ as specified in Eq. (16b).



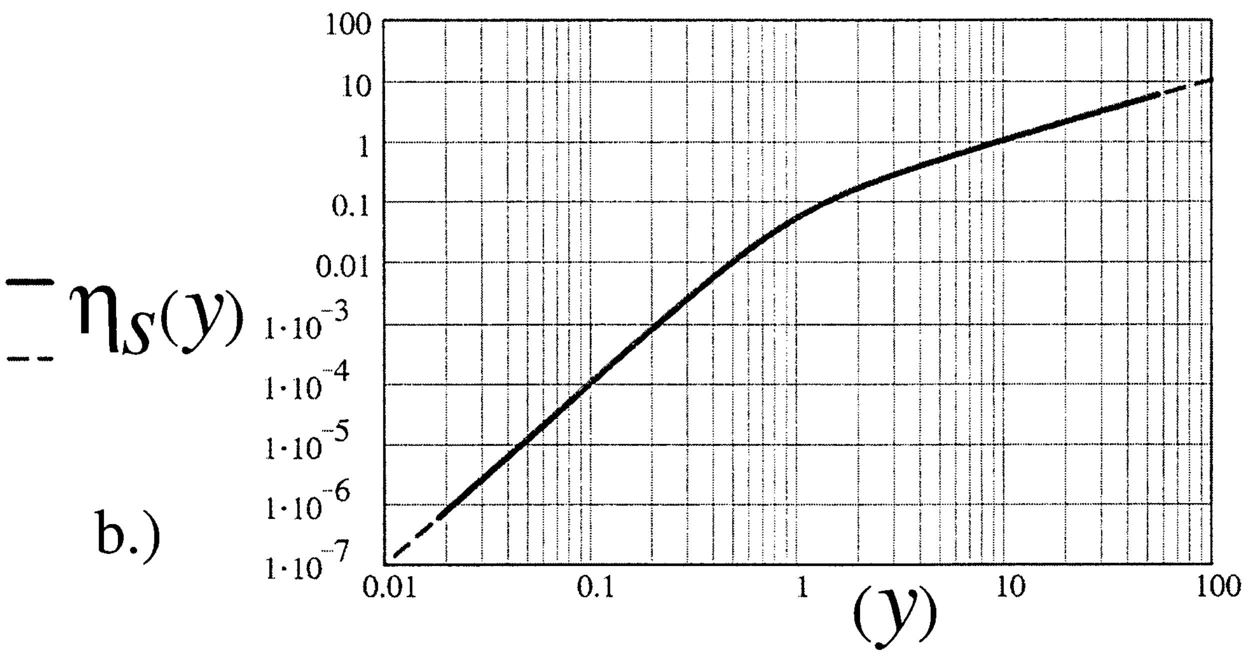


Fig. 4. Induced loss factor $\eta_s(y)$, as a function of the normalized frequency (y), resulting from replacing, adhocly, a summation by integration $[R=27;\ (M_s\,/\,M)\ =\ 10^{-1}.]$

a. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9a) and (9c), respectively.

b. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9b) and (9d), respectively.

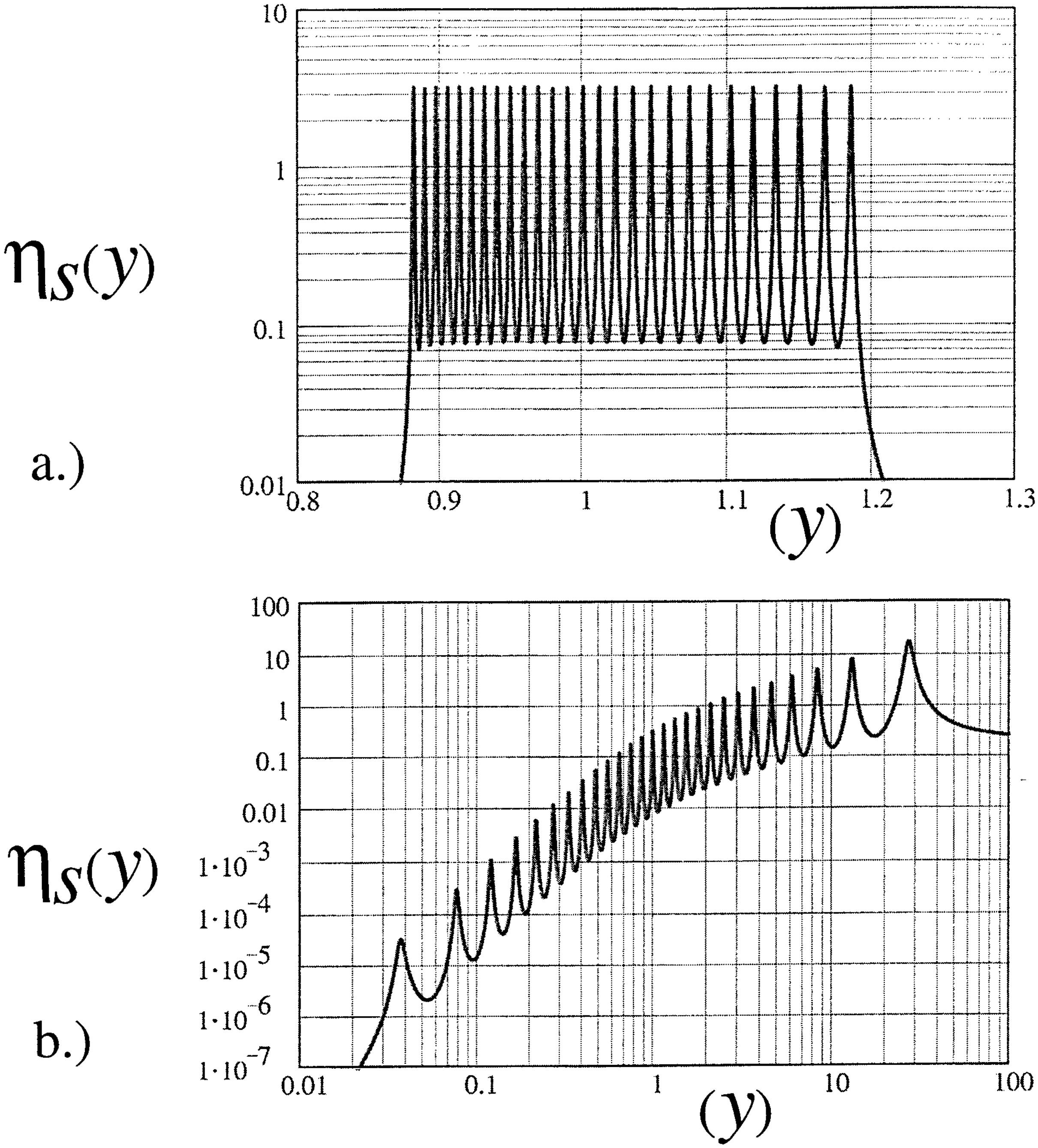


Fig. 5. Induced loss factor $\eta_s(y)$, as a function of the normalized frequency (y), for a modal overlap parameter (b_r) that is less than unity; $b_r \approx (10)^{-1}$. $[R = 27;\ (M_s / M) = 10^{-1}.]$

a. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9a) and (9c), respectively.

b. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9b) and (9d), respectively.

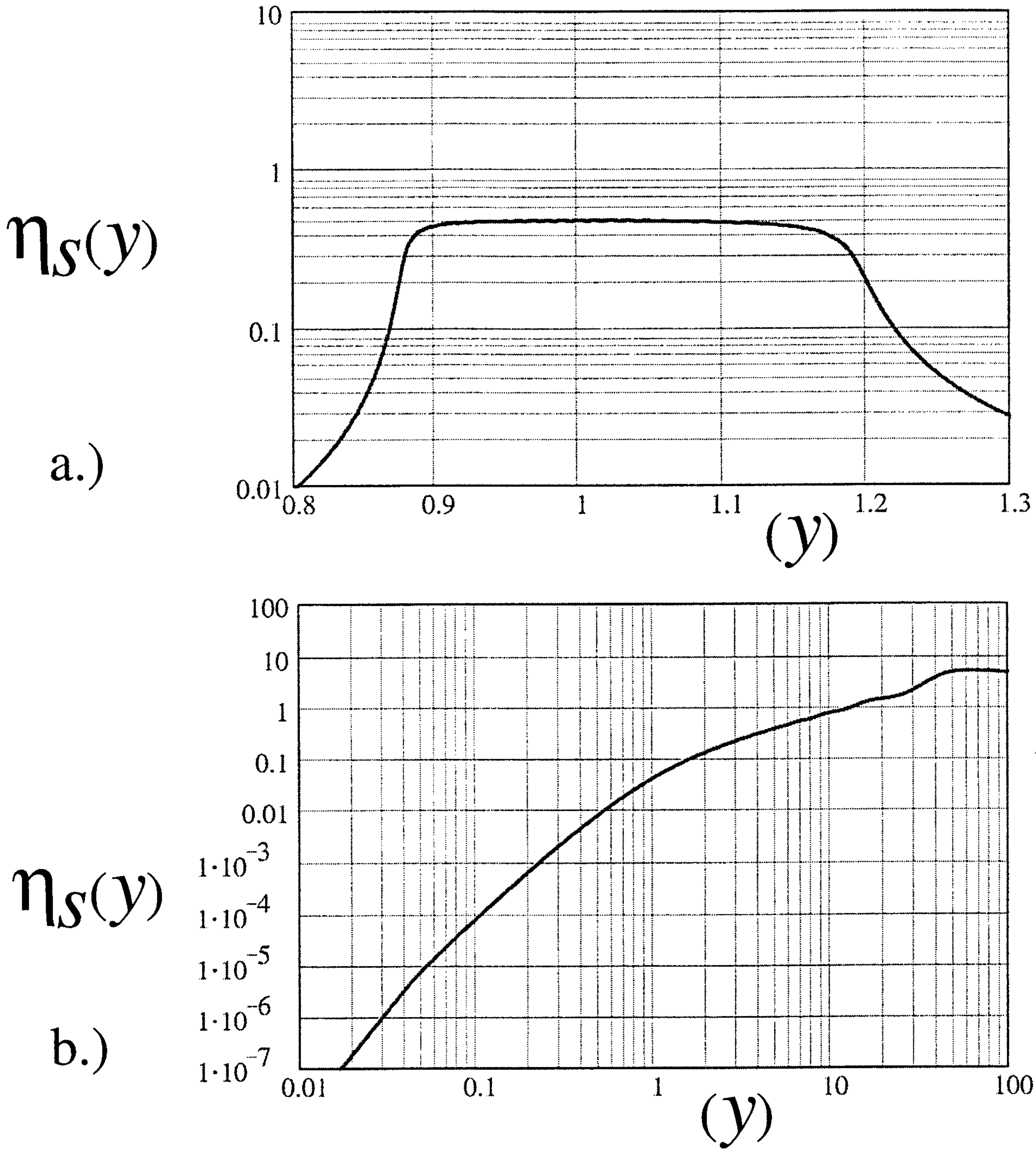
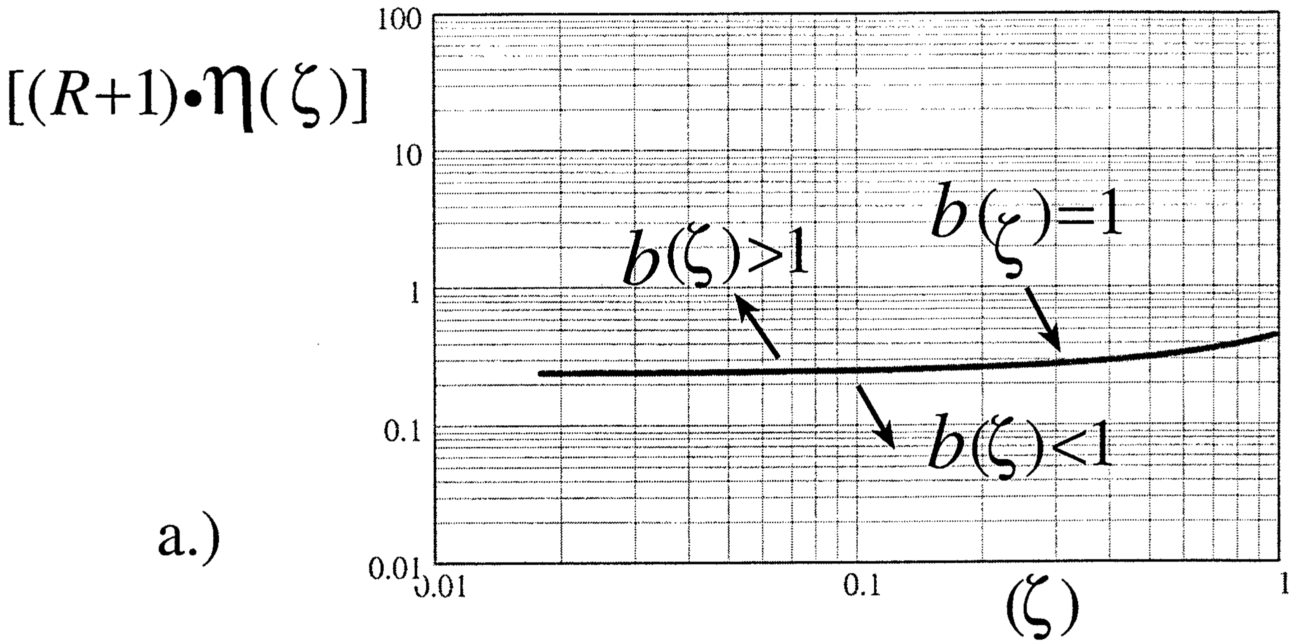


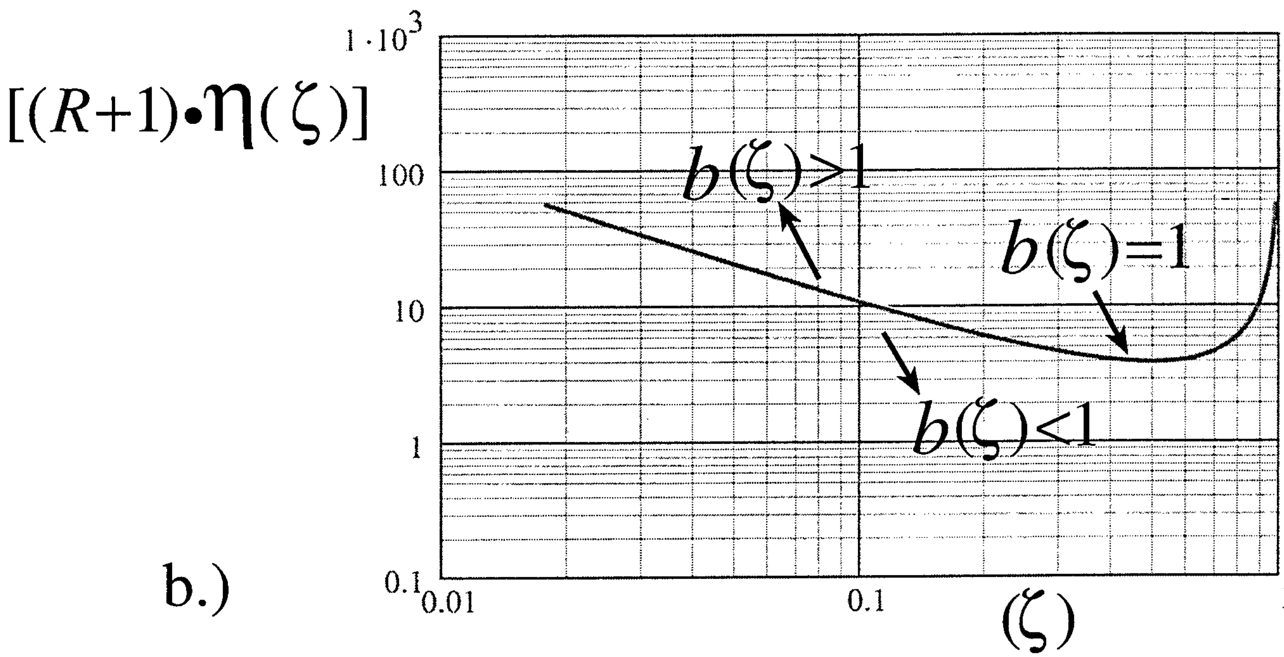
Fig. 6. Induced loss factor $\eta_s(y)$, as a function of the normalized frequency (y), for a modal overlap parameter (b_r) that exceeds unity; $b_r \approx 2$. $[R = 27;\ (M_s / M) = 10^{-1}.]$

a. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9a) and (9c), respectively.

b. For a normalized resonance frequency (x_r) and a normalized mass distribution $(\overline{m}_r)$ as specified in Eqs. (9b) and (9d), respectively.



a.)



b.)

Fig. 7. Lower limit imposed on the value of the quantity $[(R+1)\eta(\zeta)]$, as a function of (ζ), in order to just satisfy the modal overlap condition; i.e., $b_r \approx 1$, thereby, just permitting for an integration to replace a summation. [cf. Figs. 5 and 6.] $(R=27)$.

a. For a resonance frequency distribution as stated in Eq. (16a).

b. For a resonance frequency distribution as stated in Eq. (16b).

References

1. A. Pierce, V.W. Sparrow and D.A. Russell, "Fundamental structural-acoustic idealizations for structures with fuzzy internals," 1995 *Journal of Acoustics and Vibration*, **117**, 339-348.
2. M. Strasberg and D. Feit, "Vibration damping of large structures by attached small resonant structures," 1996, *Journal of the Acoustical Society of America*, **99**, 335-344.
3. R.L. Weaver, "Mean and mean-square responses of a prototypical master/fuzzy structure," 1996, *Journal of the Acoustical Society of America,* **99**, 2528-2529.
4. Yu. A. Kobelev, "Absorption of sound waves in a thin layer," 1987, *Soviet Physics Acoustics*, **33**, 295-296.
5. R.J. Nagem, I. Velijkovic and G. Sandri, "Vibration damping by a continuous distribution of undamped oscillators," 1997, *Journal of Sound and Vibration* **207**, 429-434.
6. M. Strasberg, "Continuous structure as "fuzzy" substructures," 1996 *Journal of the Acoustical Society of America*, **100**, 3456-3459.
7. G. Maidanik and K.J. Becker, "Noise control of a master harmonic oscillator coupled to a set of satellite harmonic oscillators," 1998, *Journal of the Acoustical Society of America*, **104**, 2628-2637; "Characterization of multiple-sprung mass for wideband noise control," 1999, *Journal of the Acoustical Society of America*, **106**, 3109-3118; and "Criteria for designing multiple-sprung masses for wideband noise control," 1999, *Journal of the Acoustical Society of America*, **106**, 3119-3127.
8. G. Maidanik, "Induced damping by a *nearly* continuous distribution of *nearly* undamped oscillators: Energy Analysis," 2000. In preparation.

9. G. Maidanik and J. Dickey, "Singly and regularly ribbed panels," 1988, *Journal of Sound and Vibration*, **123**, 309-314.

10. R.H. Lyon, *Statistical Energy Analysis of Dynamic Systems*, 1975, MIT, Cambridge; and R.H. Lyon and R.G. Dejung, *Theory and Application of Statistical Energy Analysis*, 1995 Butterworth-Heinemann, Boston.

11. L. Cremer, M. Heckl, and E. Ungar, *Structure-Borne Sound, Structural Vibrations and Sound Radiation at Audio Frequencies* (2nd ed.), 1988, Springer-Verlag, Berlin.

12. E, Skudrzyk, "The mean-value method of predicting the dynamic response of complex vibrators," 1980, *Journal of the Acoustical Society of America*, **67**, 1105-1135.

13. G. Maidanik, "Influence of deviation and variations in transducers on the filtering actions of spectral filters," Journal of the Acoustical Society of America, **55**, 170-183.

14. N.B. Haaser, J.P. LaSalle and J.A. Sullivan, *Introduction to Analysis*, (Volume I, Chapter 13), 1965, University of Notre Dame, GINN and Co., Boston.

INITIAL DISTRIBUTION

Copies			
2	NAVSEA 05T2		
	1	Taddeo	
	1	Biancardi	
5	ONR/ONT		
	1	334	Tucker
	1	334	Radlinski
	1	334	Vogelsong
	1	334	Main
	1	Library	
4	NRL		
	1	5130	Bucaro
	1	5130	Williams
	1	5130	Photiadis
	1	Library	
4	NUWC/NPT		
	1	Sandman	
	1	Harari	
	1	3332	Lee
	1	Library	
2	DTIC		
2	Johns Hopkins University		
	1	Green	
	1	Dickey	
2	ARL/Penn State University		
	1	Burroughs	
	1	Hwang	
1	R.H. Lyons		
	1	Lyon	
1	Cambridge Collaborative		
	1	Manning	
1	Cambridge Acoustical Associate		
	1	Garrelick	
1	J.G. Engineering Research		
	1	Greenspan	
1	MIT		
	1	Dyer	
1	Florida Atlantic University		
	1	Cuschieri	
1	Catholic Univ. of Am. Eng. Dept.		
	1	McCoy	
2	Boston University		
	1	Pierce	
	1	Barbone	
1	Penn State University		
	1	Koopman	
2	Virginia Tech		
	1	Knight	
	1	Fuller	

CENTER DISTRIBUTION

Copies	Code	Name
1	011	Corrado
2	0112	Douglas Halsall
1	20	
1	2040	Everstine
1	70	Covich
1	701	Sevik
3	7015	Fisher Vendittis
1	7020	Strasberg
1	7030	Maidanik
1	7203	Dlubac
7	7250	Shang Becker Bowers Carroll Maga Niemiec Vasudevan
1	842	Graesser
2	3421	(TIC-Carderock)